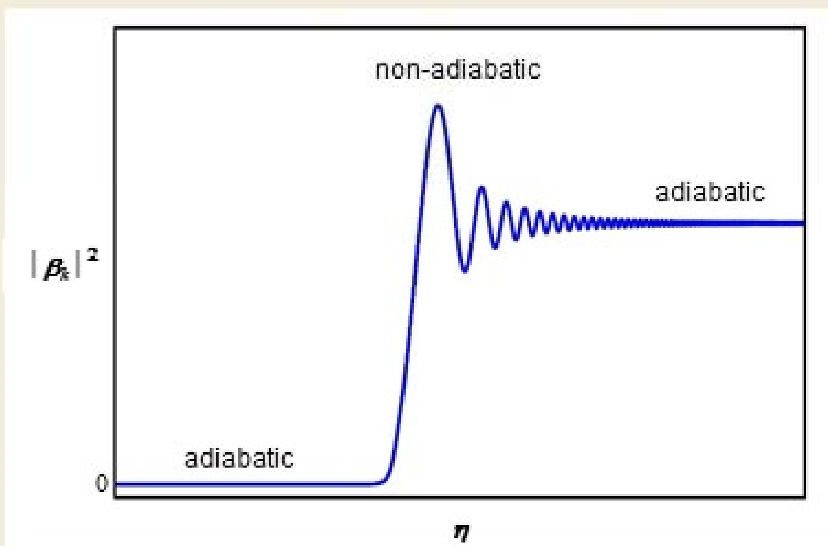
ERWIN SCHRÖDINGER'S ALARMING PHENOMENON

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

The Proper Vibrations of the Expanding Universe



Physica **VI**, 899 (1939)

Rocky Kolb
University of Chicago

November 9, 2016
Arizona State University

ERWIN SCHRÖDINGER'S ALARMING PHENOMENON

Schrödinger's 1939 deep insight into particle creation in the expanding universe

- was correct in principle (although some technical missteps)
- was not then (and isn't much now) appreciated
- was alarming to Schrödinger, but now a fundamental part of cosmology
- has profound implication for understanding our present universe
  - we are all amplified quantum fluctuations!
  - gravitons
  - dark matter?
  - other?

Lessons to be learned

- lower the anchor of our peaceful studies into the ground of eternity
- don't be alarmed
- trust your equations
- beautiful physics has far-reaching implications

THE PROPER VIBRATIONS OF THE EXPANDING UNIVERSE

by ERWIN SCHRÖDINGER

§ 1. *Introduction and summary.* Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are its proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light. — Einsteins theory of gravitation imposes an a priori reason for assuming space to be, if closed, expanding or contracting; for this theory does not admit of a stable static solution. — The observed facts are, to say the least, not contrary to these assumptions.

This makes it imperative to generalize to expanding (or contracting) universes the investigation of proper vibrations, started for the the static cases (Einstein- and De Sitter-universe) by the present writer and two of his collaborators [1]). The task is an easy one. The broad results are largely (in part even entirely) independent of the time-law of expansion. In the cases of main practical interest, i.e. with the present slow time rate of expansion and with wave lengths small compared with the radius of curvature of space (R), they are the following.

For *light*: when referred to the customary *co-moving* coordinates, an *arbitrary* wave process exhibits essentially the same succession of states as without expansion. Briefly, the wave function shares the general dilatation. Hence all *wave lengths* increase proportionally to the radius of curvature. — The *time rate* of events is slowed down. It is, in every moment, proportional to R^{-1}. Moreover all *intensities* are affected by a common factor such as to make the total energy of an arbitrary wave process proportional to R^{-1}.

For the *material particle* the broad results are these: a strictly monochromatic process (i.e. a proper vibration) again shares the

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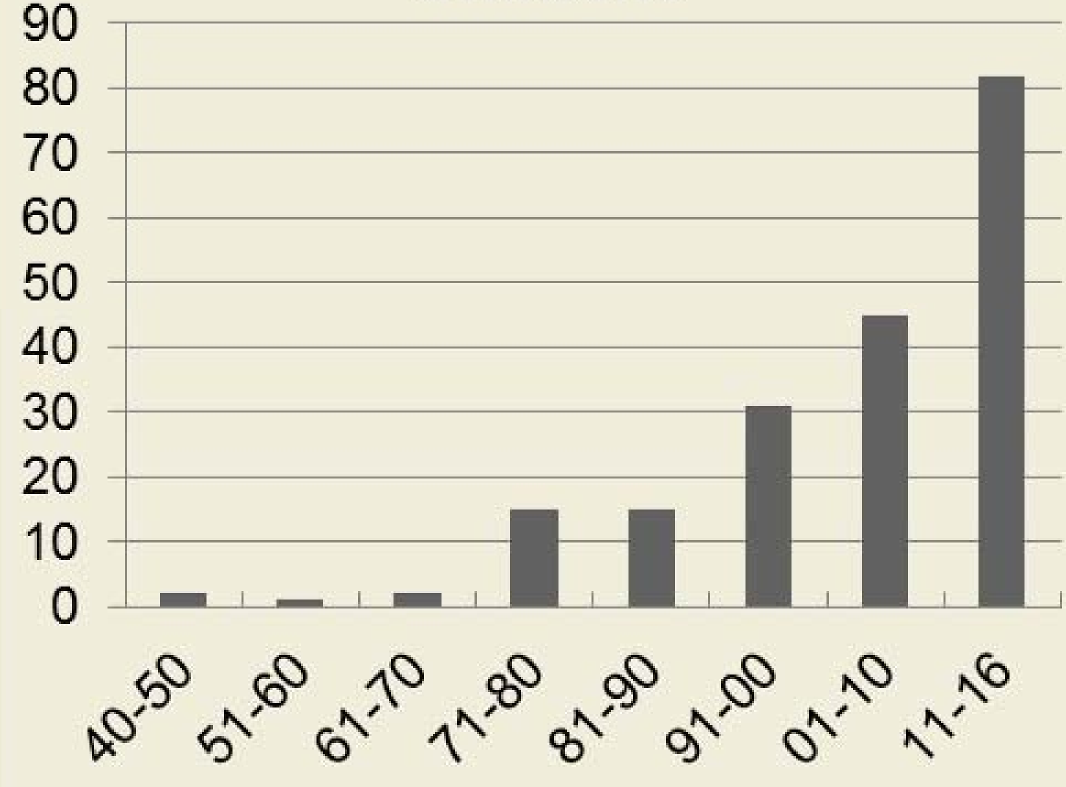

common dilatation, so that its wave length λ is proportional to R, as before. From the changing λ the changing *frequency* is calculated by de Broglies formula. This implies different frequencies to be affected by different factors. Therefore an arbitrary wave function can no longer be said to simply share the common dilatation. But since de Broglie's dispersion formula persists, the familiar connection (momentum $= h/\lambda$) between linear group velocity ($=$ particle velocity) and wave length is also preserved, which causes the former or more precisely the momentum, to decrease proportional to R^{-1}. As regards the amplitudes, the most reliable information about them, valid for any particle wave function whatsoever, is this, that the *normalisation* is rigorously conserved during the expansion.

These are the broad results. A finer and particularly interesting phenomenon is the following.

The decomposition of an arbitrary wave function into proper vibrations is rigorous, as far as the functions of space (amplitude-functions) are concerned, which, by the way, are exactly the same as in the static universe. But it is known, that, with the latter, two frequencies, equal but of opposite sign, belong to every space function. *These two* proper vibrations cannot be rigorously separated in the expanding universe. That means to say, that if in a certain moment only one of them is present, the other one can turn up in the course of time.

Generally speaking this is a phenomenon of outstanding importance. With particles it would mean production or anihilation of matter, merely by the expansion, whereas with light there would be a production of light travelling in the opposite direction, thus a sort of reflexion of light in homogeneous space. Alarmed by these prospects, I have investigated the question in more detail. Fortunately the equations admit of a solution by familiar functions, if R is a *linear* function of time. It turns out, that in this case the alarming phenomena do not occur, even within arbitrarily long periods of time. Waves travelling in one direction can be rigorously separated from those travelling in the opposite direction. The results for D'Alemberts equation (light) and Gordons equation (material particles), which have been used throughout in this paper for the sake of simplicity, are given in sect. 5 and 6 respectively. I have confirmed the results with Diracs equation, but reserve it to a subsequent paper.

Even in an expanding universe, a particle's wavefunction can be decomposed into "proper vibrations" (positive & negative frequency modes):

$$\Psi(t) = \frac{\alpha}{\sqrt{2\omega}}\, e^{-i\omega t} + \frac{\beta}{\sqrt{2\omega}}\, e^{+i\omega t}$$

Particle occupancy number $\propto |\beta|^2$

If start with pure incoming or outgoing waves, in and out will become mixed.

The expansion of the universe creates particles!

This alarms me [ed. why?], so I wrote a paper.

$e^{2\pi i \nu t}$ will re-assume (or approximately re-assume) the form $Ae^{2\pi i \nu' t}$ — and *not* $Ae^{2\pi i \nu' t} + Be^{-2\pi i \nu' t}$ — whenever $R(t)$, after an intermediate period of arbitrary variation, returns to constancy (or to approximate constancy). I can see no reason whatsoever for $f(t)$ to behave rigorously in this way, and indeed I do not think it does. There will thus be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to what in the introduction I called „the alarming phenomena". They are certainly very slight, though, in two cases, viz. 1) when R varies slowly 2) when it is a linear function of time (see the following sections).

A second remark about the new concept of proper vibration is, that it is not always invariantly determined by the form of the universe. The separation of time from the spatial coordinates may succeed in a number of different space-time-frames. For De Sitters universe I know three of them. Besides the static one, for which P. O. Müller (l.c.) has redently given the proper vibrations, there is an expanding form with infinite R and an expanding form with finite R *). A proper vibration of one frame will not transform into a proper vibration of the other frame, for the separation of variables is destroyed by the transformation.

---

*) From De Sitters line-element in static form

$$ds^2 = -R^2_0[d\chi^2 + \sin^2\chi(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)] + R^2_0 \cos^2\chi\, dt^2$$

the transformation of Lemaitre (J. Math. and Phys. M.I.T., 4, 188, 1925) and Robertson (Phil. Mag. 5, 835, 1928)

$$\bar{r} = R_0 \operatorname{tg}\chi\, e^{-t} \quad \bar{t} = t + \lg\cos\chi$$

gives the expanding *flat* form

$$ds^2 = -e^{2\bar{t}}[d\bar{r}^2 + \bar{r}^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)] + R^2_0 d\bar{t}^2.$$

The following transformation

$$\operatorname{tg}\chi' = \frac{\operatorname{tg}\chi}{\operatorname{Cos} t} \quad \operatorname{Sin} t' = \operatorname{Sin} t \cos\chi$$

or

$$\sin\chi = \sin\chi' \operatorname{Cos} t' \quad \operatorname{Tg} t = \operatorname{Tg} t'(\cos\chi')^{-1}$$

gives the expanding *curved* form

$$ds^2 = -R^2_0(\operatorname{Cos} t')^2[d\chi'^2 + \sin^2\chi'(d\vartheta^2 + \sin^2\vartheta d\varphi^2)] + R^2_0 dt'^2.$$

(In this footnote R_0 is a constant length and the cosmical times t, $\bar{t}$, t' are dimensionless.)

Schrödinger's two favorite phrases:

1. alarming phenomenon
2. mutual adulteration

Schrödinger was alarmed by creation of a *single* particle

per Hubble time ($H_0^{-1} \sim 10^{10}$ yr)

per Hubble volume ($H_0^{-3} \sim 10^{57}$ km^3)

with Hubble energy ($H_0 \sim 10^{-33}$eV)

Schrödinger's Alarming Times

His private life seemed strange to bourgeois people like ourselves. But all this does not matter. He was a most lovable person, independent, amusing, temperamental, kind and generous, and he had a most perfect and efficient brain.

Max Born on Schrödinger's "private life"

Schrödinger's Alarming Times

1926: "Quantisierung als Eigenwertproblem," *Annalen der Physik*. **384**, 273

1927: Schrödinger visits U.S.

Found noise and dirt of New York "shattering"

Found Chicago worse, feared "bandits who spring with loaded guns from speeding autos." (Anny liked Chicago.)

Schrödinger departs UZH for Berlin.

1933: Nobel Prize. Nazis come to power. Schrödinger, marked by Nazis as "politically unreliable," departs Berlin for "exile" in Oxford.

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Biographical info. from Walter Moore, *Schrödinger, Life and Thought* (Cambridge Univ. Press, 1992)

Schrödinger – Fermi

http://www.shardcore.org/

http://www.theflorentine.net/

Dublin
February 10, 1951

Dear Fermi,

….. I beg you to help me remove once and for all, a remorse that I cannot help associating with my memory of you at our last meeting, namely that I still owe you Lire 400 val. Sept 1938. To re-calculate this sum to date, now that all money-value has gone down is very difficult, but I think something like 200 Swedish Crowns would be a modest estimate for re-payment. If you agree and if you still have an account at Stockholm, this would be very simple. If the later is not the case, please indicate me your bankers' account at Chicago, and I hope to manage even so.

…..

Yours very sincerely,
E. Schrödinger

http://www.shardcore.org/

Schrödinger – Fermi

http://www.theflorentine.net/

Chicago
February 27, 1951

Dear Shrodinger [sic],

….. As to the old debt that you mention, I believe that you are estimating the value of 400 lire too high. At that time the lire was worth about one twentieth of one dollar and it seems therefore a $20.00 settlement would be correct. I no longer have an account in Sweden. My bank here in Chicago is the University National Bank, 1354 East 55th Street, Chicago 15. Please however, be sure if there are any difficulties whatsoever about transferring this amount not to worry about it because it is certainly not worth it.

…..

Yours very sincerely,
Enrico Fermi

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Schrödinger the Cosmologist

LETTERS TO THE EDITORS

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NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 600.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Nature of the Nebular Red-Shift

FROM an investigation (to be published in *Physica*) of the proper vibrations of expanding spherical space, it follows that—in extremely good approximation—light is propagated with respect to co-moving co-ordinates irrespective of the expansion, except that (*a*) the time-rate of events is slowed down and (*b*) all energy portions decrease, both inversely proportional to the radius of curvature.

The slowing down secures the constancy of the velocity of light and entails the nebular red-shift, which from this point of view takes place *during the passage*. The attempt[1] to decide by observation, whether it is actually due to expansion, rests on two important formulæ, which follow from the new view with great ease. Let l be the linear diameter of a nebula *at the moment of emission* and χ its angular distance from the observer (linear distance divided by the circumference of space), then the angle $d\theta$ between two geodesics of space, pointing at the moment of emission from the observer to the ends of the diameter, is from pure geometry:

$$d\theta = \frac{l}{R \sin \chi}, \qquad (1)$$

R being the radius of curvature at the moment of emission. By the theorem quoted above, $d\theta$ is also the observed angular diameter of the nebula (Hubble and Tolman, equation 3).

Again, let the energy emitted by the nebula within an appropriately chosen unit of time be E_s. It will soon assume the shape of a spherical shell of thickness c (say). Let $R_{obs.}$ be the radius of space, when this shell reaches the observer. Its surface at this moment is, by pure geometry, $4\pi R^2_{obs.} \sin^2\chi$. By the theorem quoted above, its thickness then is $c\, R_{obs.}/R$ and its energy is $E_s\, R/R_{obs.}$. Hence its energy density ρ is

$$\rho = \frac{E_s}{4\pi c\, R^4_{obs.}} \cdot \frac{R^2}{\sin^2\chi}. \qquad (2)$$

ρ is a measure of the bolometric luminosity, observed outside the earth's atmosphere (Hubble and Tolman, equation 4).

My purpose in re-stating here these two important formulæ due to Tolman is to make the following remarks. Both l and E_s refer to the moment of emission, which is different for two nebulæ observed simultaneously. Should l and E_s exhibit a general dependence on R, then it would no longer be reasonable to regard them as *constants*, when equations (1) and (2) are combined (as they actually are) with the hypothesis of uniform spatial distribution of the nebulæ. For the latter, if admitted at all, has to apply to nebulæ which are intrinsically similar at the *same* moment of time—not at such moments as depend on the accidental position of our galaxy.

As regards l, the question is, whether we are inclined to assume (*a*) that the distances between the stars within a nebula behave, on the average, like the distances between two points of a rigid body—say, the ends of the Paris metre rod; or (*b*) like the distance between two distant nebulæ. Clearly the case of the stars is intermediate. To regard l as a constant means to decide for the first alternative. The second one would make l/R constant, giving formula (1) the same form as in the case of a non-recessional explanation of the red-shift (see Hubble and Tolman, equation 3').

As regards E_s, the possible general decline of the nebular candle-powers has already been mentioned by Hubble and Tolman (see their concluding remarks). To the assumption that the same amount of energy is emitted during every second, there is a peculiarly simple alternative, namely, that the amounts of energy, which *have* been emitted during a second, *remain* equal. On account of the decay of travelling energy, this assumption would mean $E_s \sim 1/R$, which reduces equation 2 to the same form as in the case of a non-recessional explanation of the red-shift (see Hubble and Tolman, equation 4'). I do not mean to suggest $E \sim 1/R$ particularly. I mention it in the way of an example.

These remarks detract nothing from the importance of deciding by observation how $d\theta$ and ρ actually behave, if the photographs are interpreted as assuming uniform spatial distribution. I understand that present evidence points to observed luminosities (ρ) decreasing with distance *not even quite as rapidly* as we should expect (with E_s = const.) from the non-recessional explanation. If that is so, I should say they rather support the recessional explanation, in spite of its predicting a still more rapid decrease of the ρ's. The discrepancy, though greater, can here be removed by assuming the E_s's to decrease with time; an assumption which is very plausible in an expanding universe, which, on the whole, cools down; but not at all plausible in a static one.

E. SCHRÖDINGER.

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The Forbidden $^3P_2$–$^1D_2$ Line of O III in the Nebular Spectrum of Nova Herculis 1934

ALTHOUGH the two well-known lines of [O III] $\lambda = 5007$ A. ($^3P_2$–$^1D_2$) and $\lambda = 4959$ A. ($^3P_1$–$^1D_2$) are the most prominent features in the spectra of planetary nebulæ and novæ at the nebular stage, the third line of the triplet, corresponding to the $^3P_0$–$^1D_2$

In Belgium Schrödinger met cosmologist Abbé Georges Lemaître.

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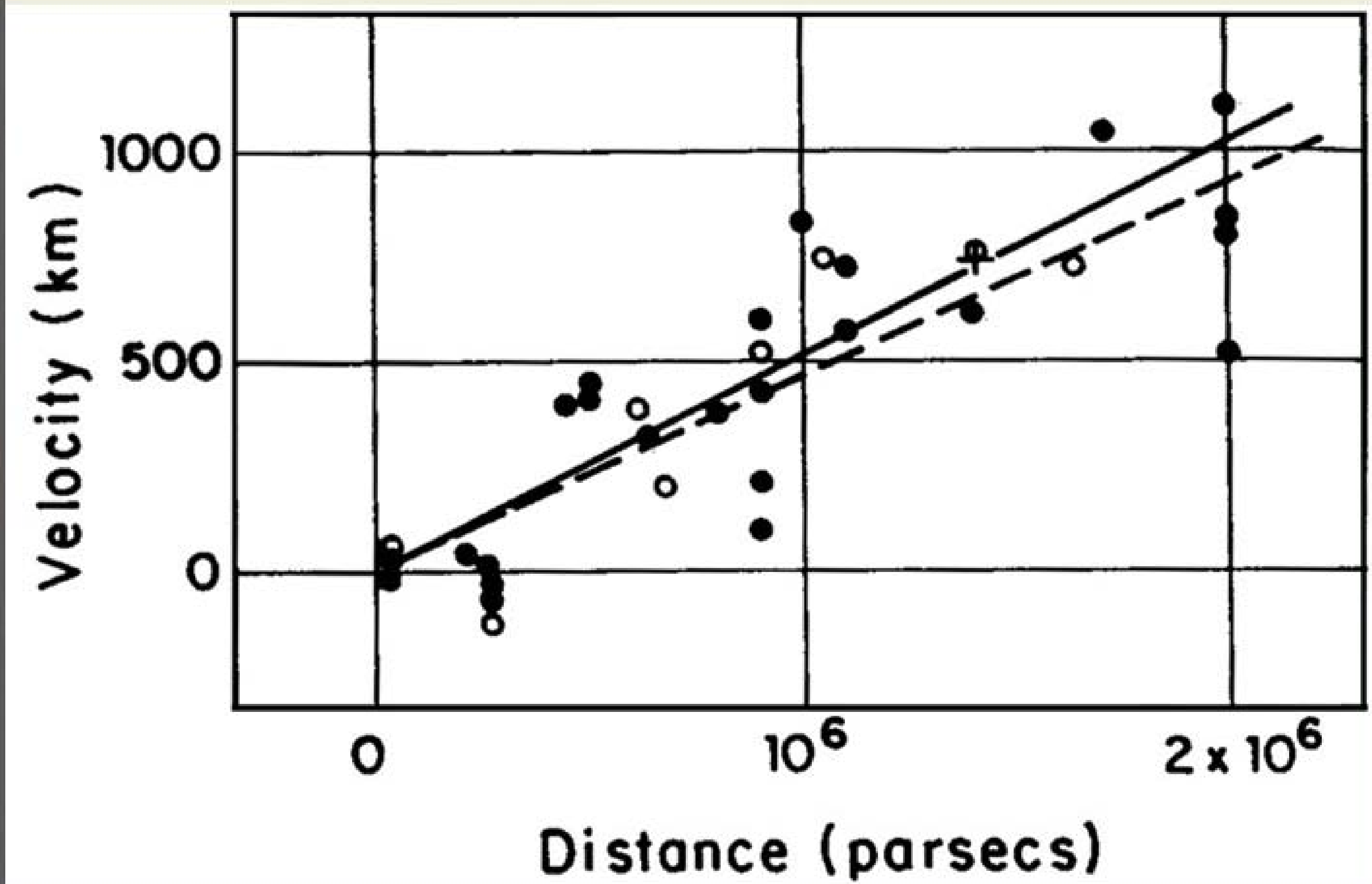
Expansion of the Universe



Hubble's discovery paper

Proc. Natl. Acad. Sci. **15**, 168 (1929)

Schrödinger the Cosmologist

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Kepler’s Alarming Times

When the storm rages and the state is threatened by shipwreck, we can do nothing more noble than to lower the anchor of our peaceful studies into the ground of eternity.

– Johannes Kepler

Schrödinger's Alarming Phenomenon

Why was Schrödinger alarmed?

How to understand mutual adulteration (particle creation)?

Why is it important?

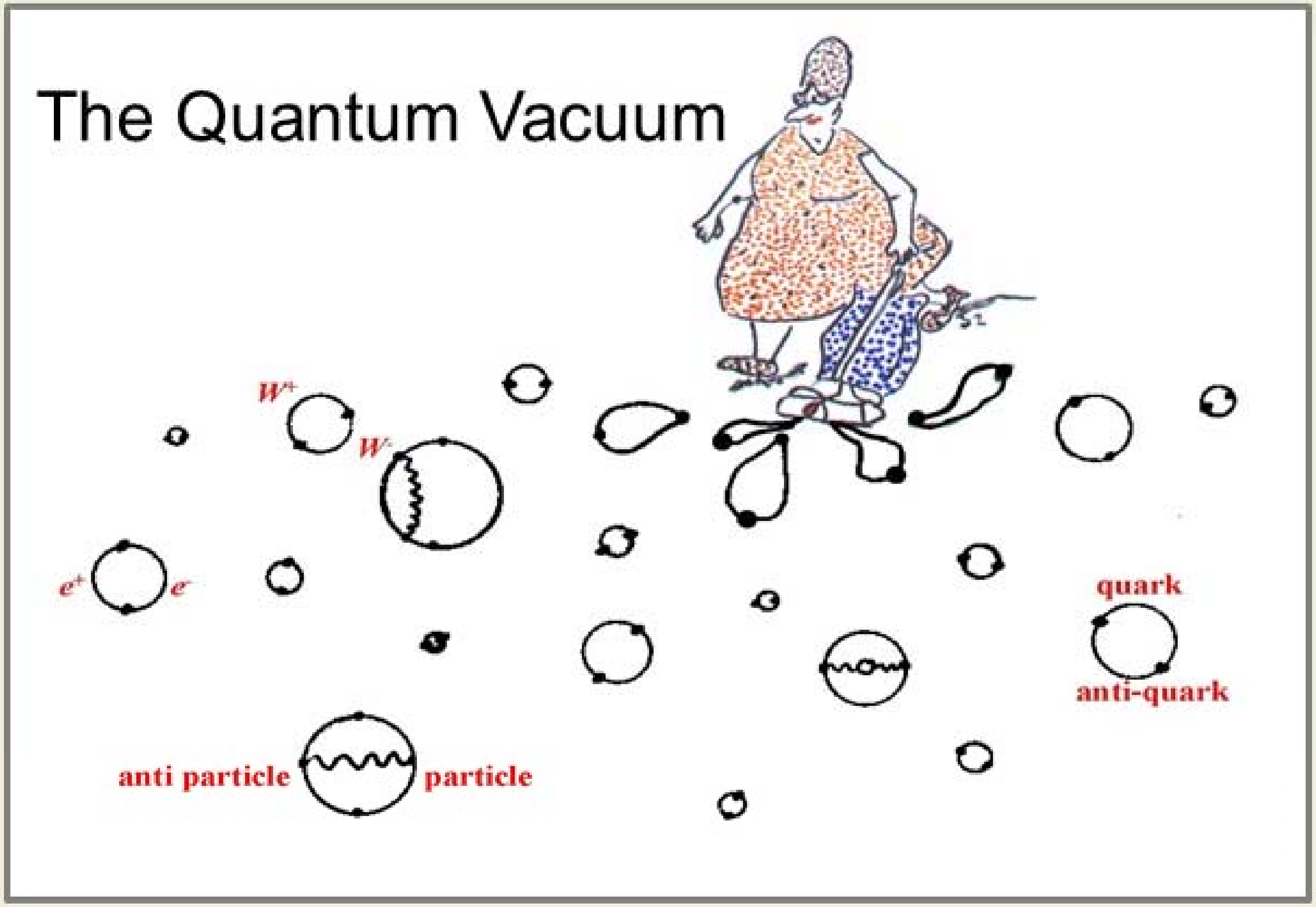
Schrödinger's Alarming Phenomenon

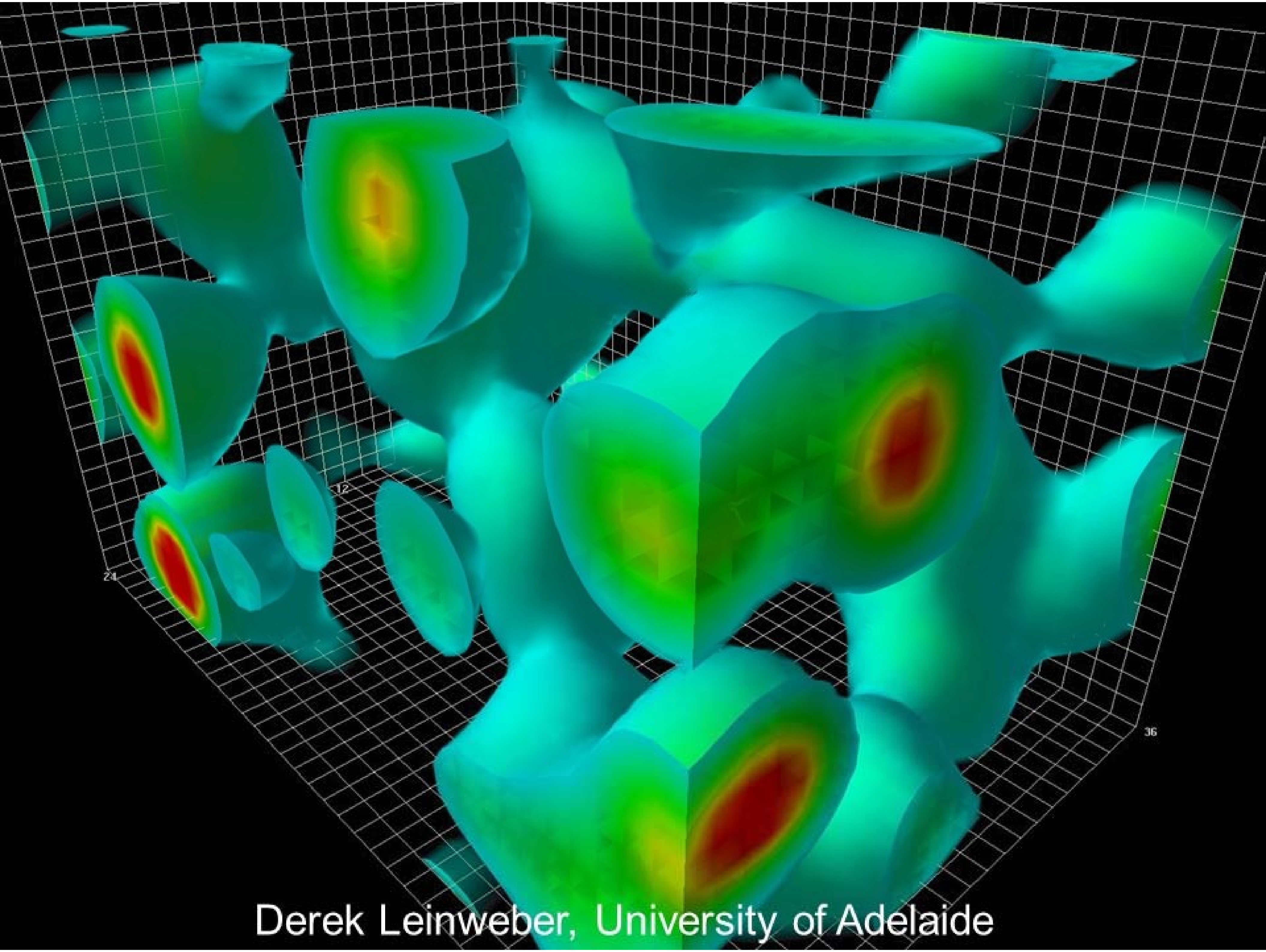
Why was Schrödinger alarmed?

- Appearance of particles from the vacuum sounds crazy.
- Technical issues with calculation:
  Only create particles with mass less than expansion rate H
  (today $H_0 \sim 10^{-33}$ eV).
  Only create particles if violate Weyl Conformal Invariance
  (don't create photons).
  Would Schrödinger still have been alarmed?
- Schrödinger looked for (and found) a cosmological solution without mutual adulteration (not a very physical solution).
- Perhaps he thought it was conceptual challenge to Quantum Mechanics, Quantum Field Theory, or General Relativity.
- Infinite particle creation in standard big-bang at $t = 0$.
- (Sometimes should just follow the equations).

Schrödinger's Alarming Phenomenon

How to understand mutual adulteration (particle creation)?

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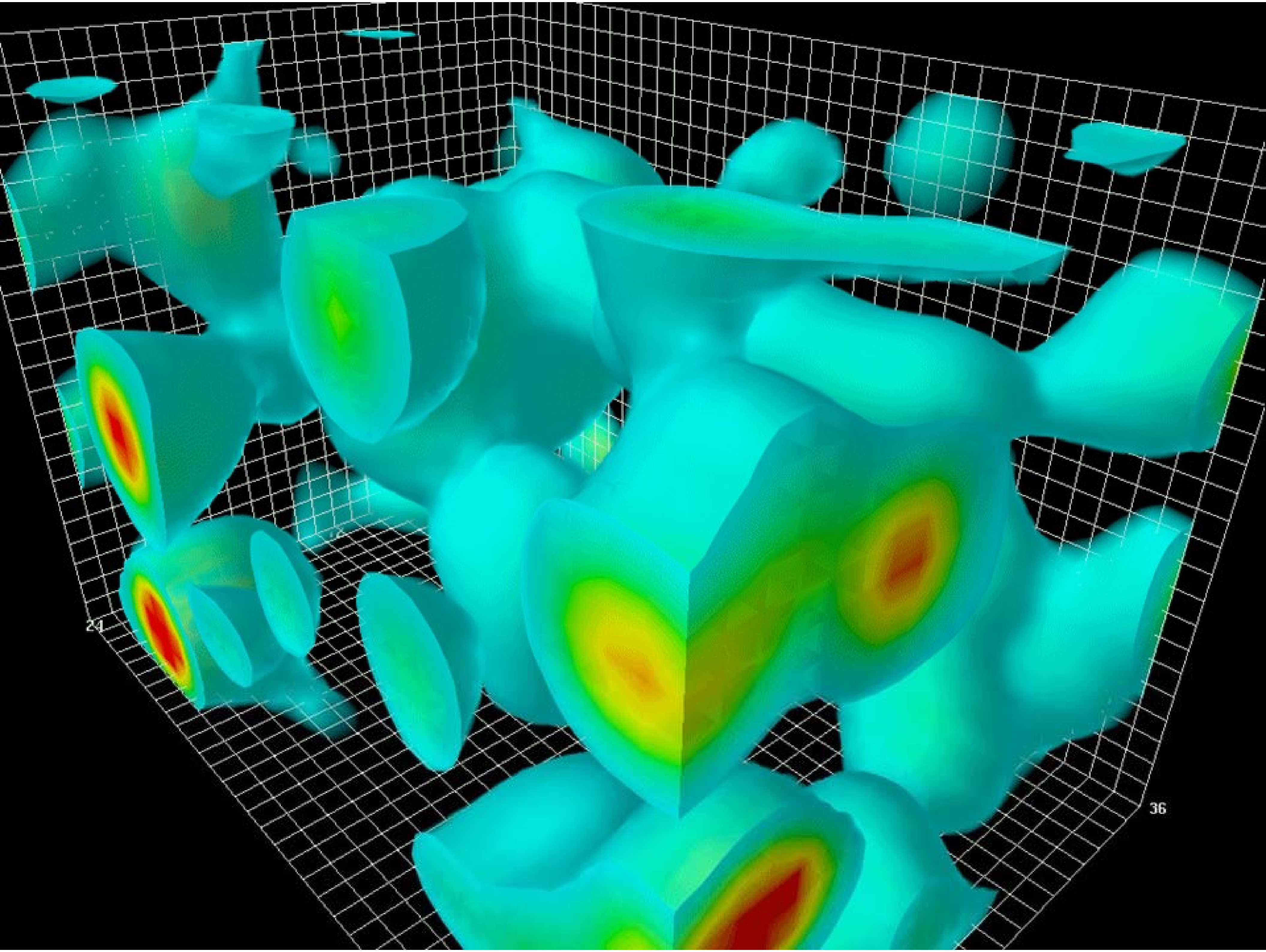
Derek Leinweber, University of Adelaide



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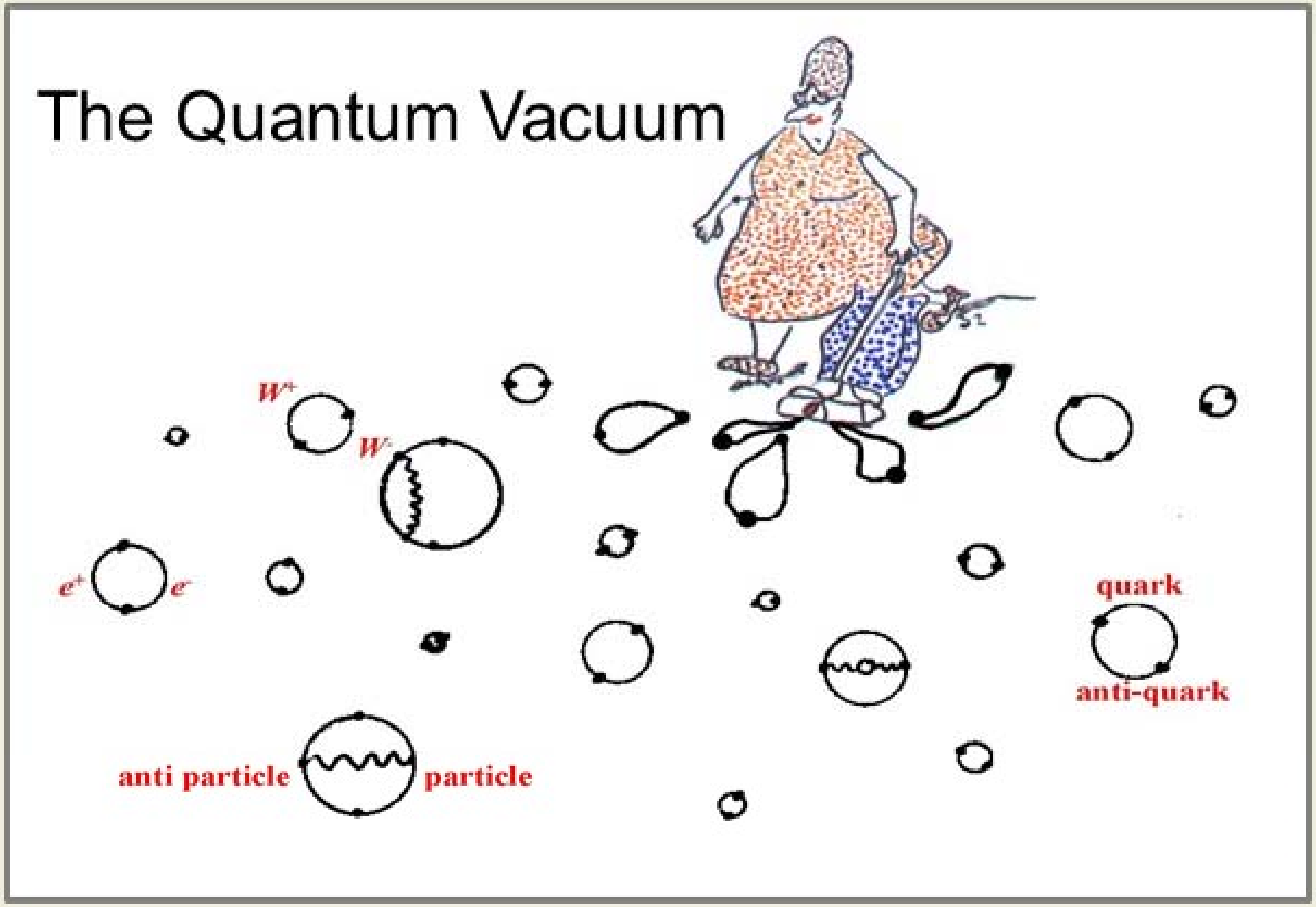
Schrödinger's Alarming Phenomenon

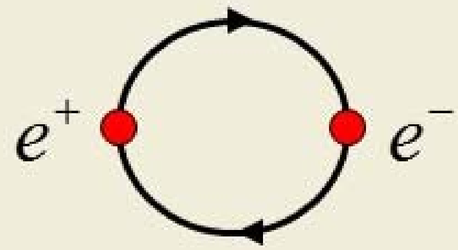
How to understand mutual adulteration (particle creation)?

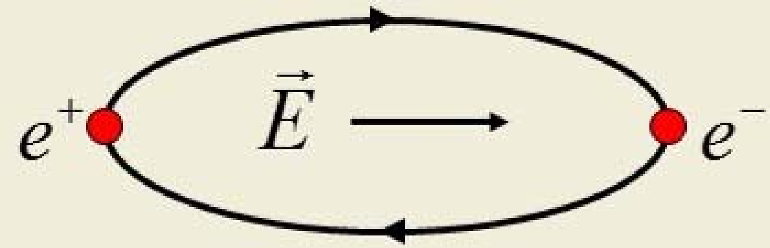


External Fields Can Disturb The Quantum Vacuum

Disturbing the Quantum Vacuum

Changing Electric Field ⟶ Particle creation



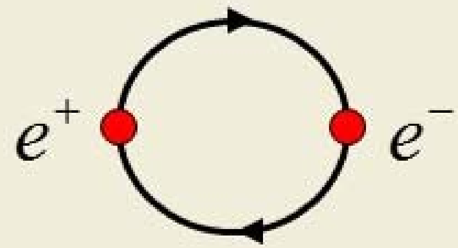


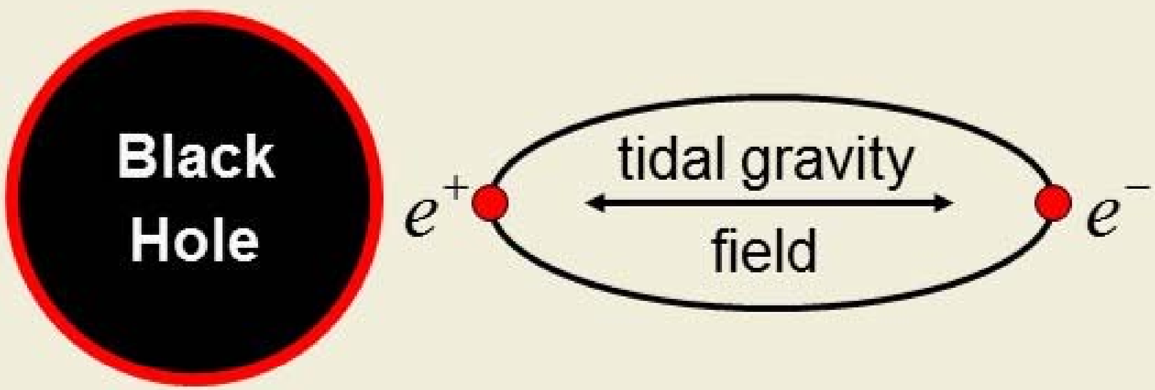
Particle creation if energy gained in acceleration from electric field over a Compton wavelength exceeds the particle's rest mass.

Heisenberg & Euler (1935); Weisskopf (1936); Schwinger (1951)

Disturbing the Quantum Vacuum

Tidal gravitational field → Particle creation



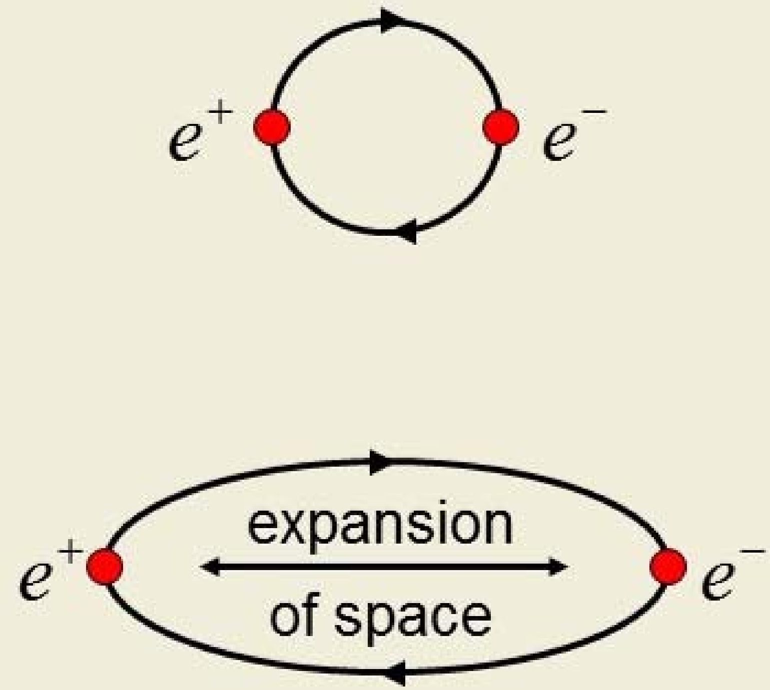


Particle creation if energy gained in acceleration from gravitational field over a Compton wavelength exceeds the particle's rest mass.

Hawking (1974); Bekenstein (1972)

Disturbing the Quantum Vacuum

Expanding universe ⟶ Particle creation

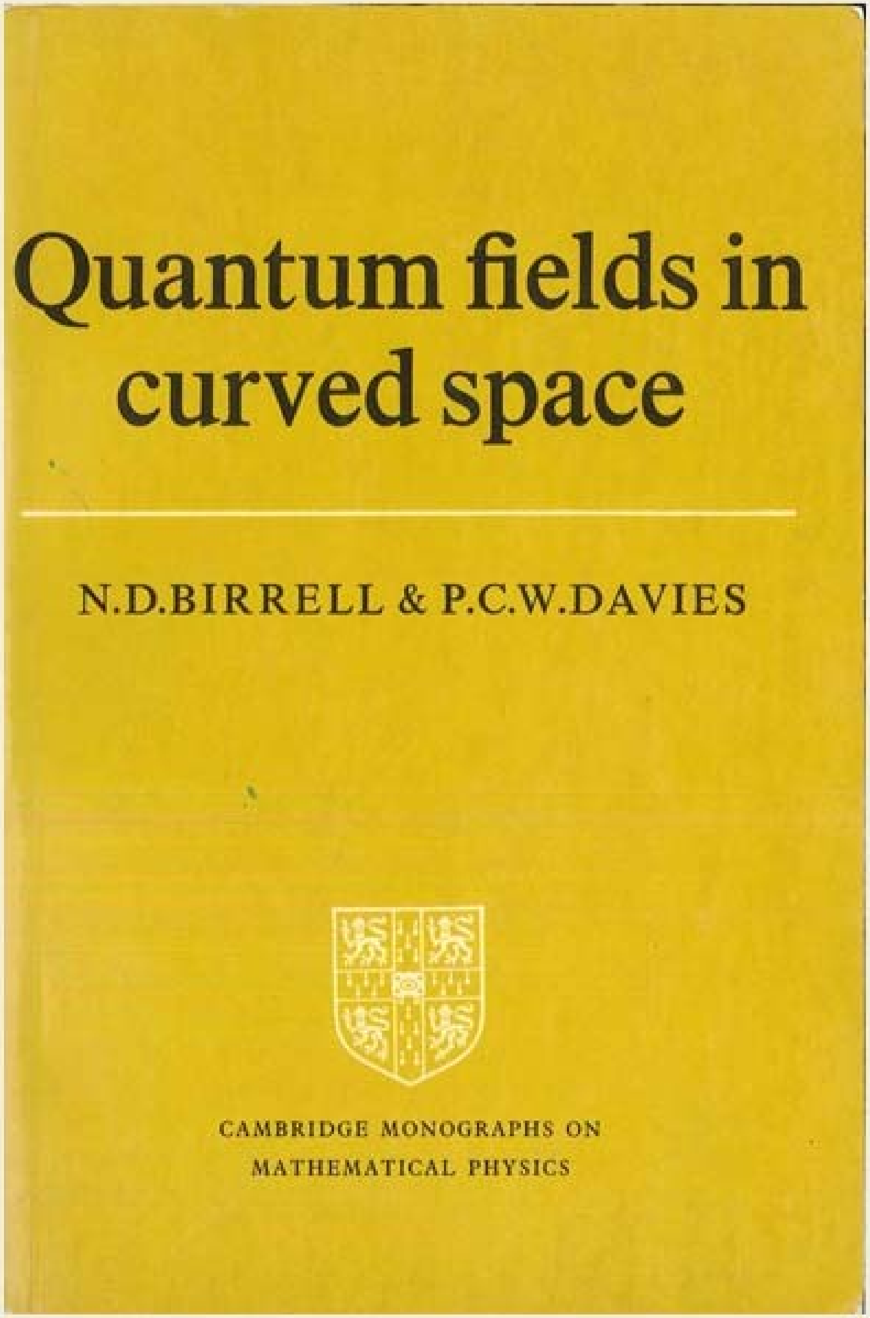


Particle creation if energy gained in expansion over a Compton wavelength exceeds the particle's rest mass.

Schrödinger's Alarming Phenomenon (1939)

Schrödinger's Alarming Phenomenon

Technical Details:



Schrödinger's Alarming Phenomenon

Free quantum scalar field χ of mass M in Minkowski space:

$$S = \int d^4x \left[\tfrac{1}{2} \partial^\mu \chi \partial_\mu \chi - \tfrac{1}{2} M^2 \chi^2 \right]$$

Mode expand and make plane-wave ansatz:

$$\chi(x,t) = \sum_k \hat{a}_k \, e^{ik\cdot x} \chi_k(t) + \hat{a}_k^\dagger \, e^{-ik\cdot x} \chi_k^*(t)$$

Equation of motion (Klein-Gordon equation):

$$\ddot{\chi}_k(t) + \omega_k^2 \chi_k(t) = 0 \qquad\qquad \omega_k^2 = \left|\vec{k}\right|^2 + M^2$$

Choose pure outgoing (+ frequency) solution

$$\chi_k(t) = \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t}$$

other solution: $\chi_k(t) = \frac{1}{\sqrt{2\omega_k}} e^{+i\omega_k t}$

Schrödinger's Alarming Phenomenon

Couple scalar field χ to gravity:

$$S=\int d^4x\sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\tfrac{1}{2}M^2\chi^2-\xi R\chi^2\right]$$ $g_{\mu\nu}$ is the metric tensor

Cosmological background, metric one function, scale factor $a(t)$

Hubble parameter, $\dot{a}(t)/a(t)=H(t)$ expansion rate of the universe

Equation of motion:

$$\ddot{\chi}_k(t)+\frac{\dot{a}(t)}{a(t)}\dot{\chi}_k(t)+\left[\frac{1}{a^2(t)}\left|\vec{k}\right|^2-(1-6\xi)\left[\frac{\dot{a}^2(t)}{a^2(t)}+\frac{\ddot{a}(t)}{a(t)}\right]+M^2\right]\chi_k(t)=0$$

If express in terms of conformal time η, $d\eta\equiv a^{-1}(t)\,dt$:

$$\chi_k''(\eta)+\omega_k^2(\eta)\chi_k(\eta)=0 \qquad \omega_k^2(\eta)=\left|\vec{k}\right|^2-(1-6\xi)\frac{a''(\eta)}{a(\eta)}+a^2(\eta)M^2$$

wave equation with <u>time-dependent</u> mass that depends on evolution in time of the scale factor

Schrödinger's Alarming Phenomenon

Solutions to wave equation are adulterated:

$$\chi_k''(\eta)+\omega_k^2(\eta)\,\chi_k(\eta)=0 \qquad\qquad \omega_k^2(\eta)=\left|\vec{k}\right|^2-(1-6\xi)\frac{a''(\eta)}{a(\eta)}+a^2(\eta)M^2$$

Pure outgoing (+ frequency) solution $\chi_k(\eta)=\dfrac{1}{\sqrt{2\omega_k(\eta)}}e^{-i\int \omega_k(\eta)d\eta}$

is a good solution if $\left|\dfrac{\omega_k'(\eta)}{\omega_k^2(\eta)}\right|^2 \ll 1$ and $\left|\dfrac{\omega_k''(\eta)}{\omega_k^3(\eta)}\right|^2 \ll 1$

Abrupt changes in $a(\eta)$ leads to abrupt changes in $\omega_k(\eta)$, which *adulterates* positive and negative frequency modes, leading to *Schrödinger's Alarming Phenomenon* of particle creation in the expanding universe.

Schrödinger's Alarming Phenomenon

Solutions to wave equation include both + and – frequency terms

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{-i\int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{+i\int \omega_k(\eta) d\eta}$$

If start with only outgoing waves, $\beta_k(\eta) = 0$,
generate incoming waves, $\beta_k(\eta) \neq 0$.

Comoving number density of particles at late time is

$$n = \frac{1}{(2\pi)^3} \int d^3\vec{k} \, \left|\beta_k(\eta)\right|^2$$

Schrödinger's Alarming Phenomenon

- Expansion of the Universe leads to time dependence of the coupling of fields to gravity

- Expansion of the universe leads to creation of all species of particles so long as there is a "time" dependence to $\omega_k(\eta)$

$$\omega_k^2(\eta) = \left|\vec{k}\right|^2 - (1-6\xi)\frac{a''(\eta)}{a(\eta)} + a^2(\eta)M^2$$ note: $\frac{a''(\eta)}{a(\eta)}$ can be positive

- Adulteration efficacy depends on abruptness of change in $\omega_k(\eta)$

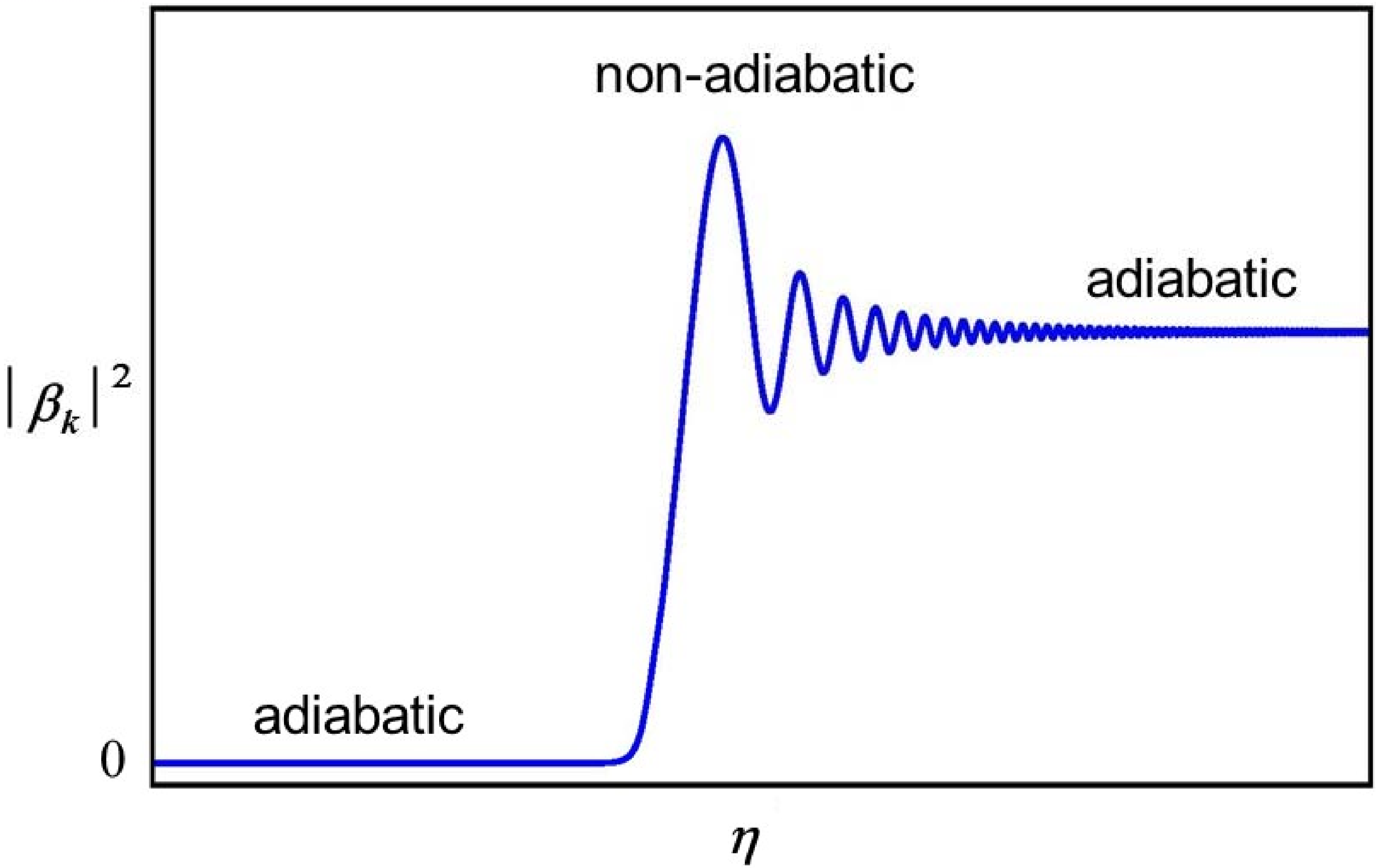
$$\left|\frac{\omega_k'(\eta)}{\omega_k^2(\eta)}\right|^2 \text{ and } \left|\frac{\omega_k''(\eta)}{\omega_k^3(\eta)}\right|^2$$

production suppressed if $M \gtrsim H$

particles "produced" when $\left|\vec{k}\right|/a \simeq H$ $(\xi = 0)$

- Many subtleties glossed over

Schrödinger's Alarming Phenomenon

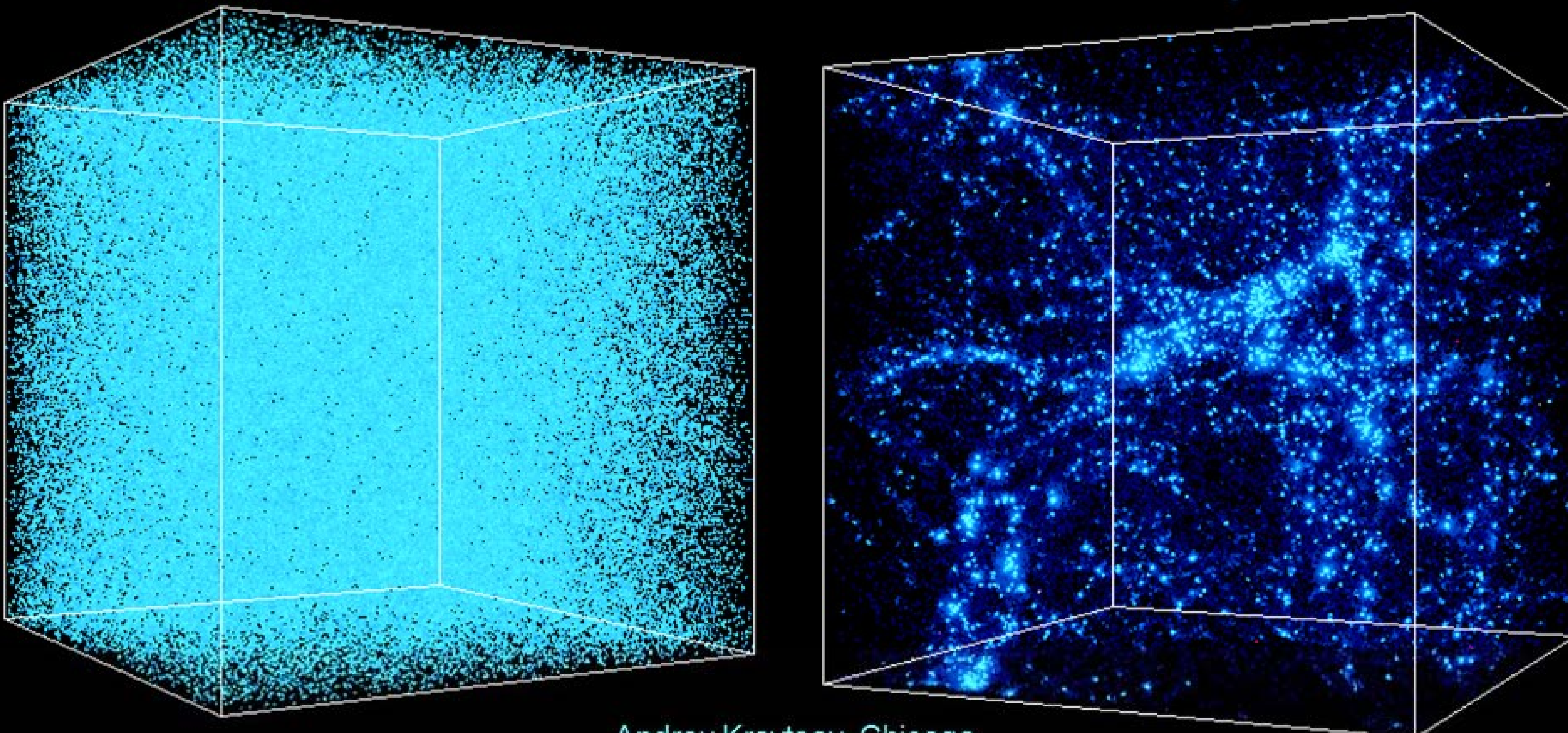


Schrödinger's Alarming Phenomenon

Why is it important?

Primordial Seeds of Structure

Primordial Universe | Today

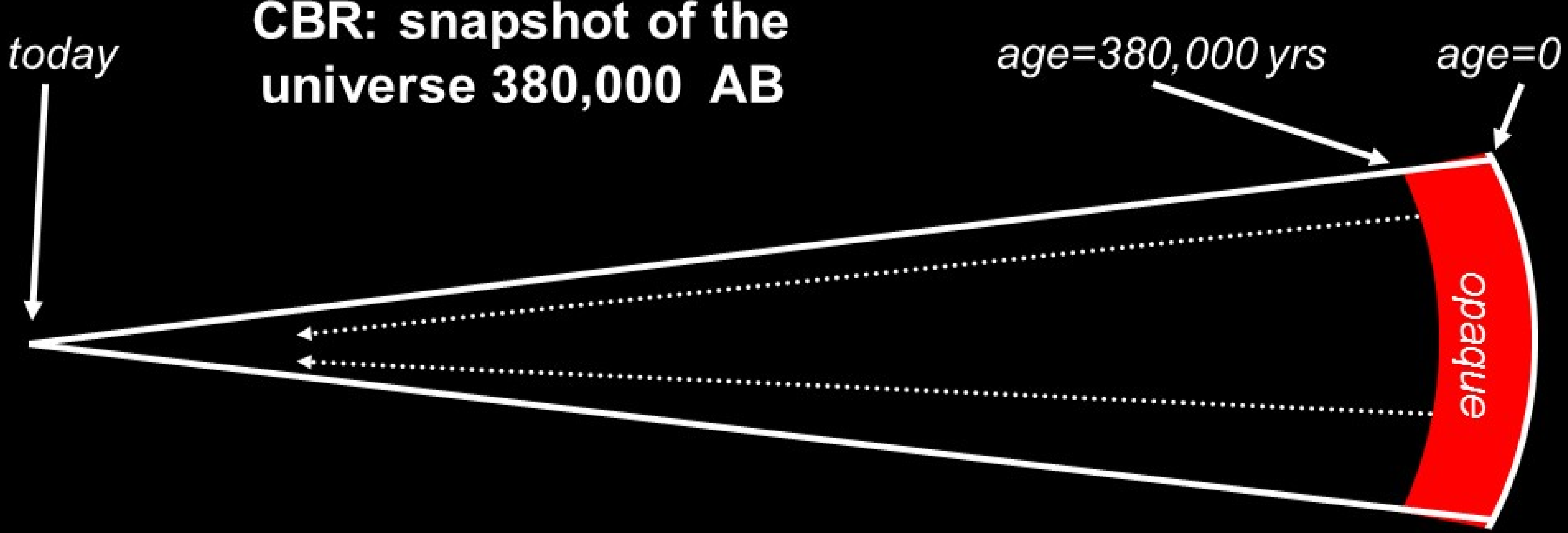


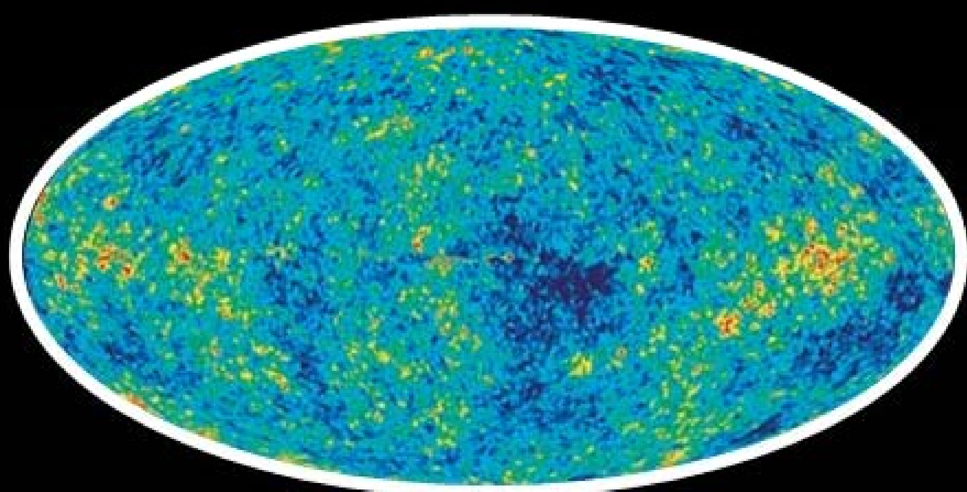
Andrey Kravtsov, Chicago

If you can look into the seeds of time
And say which grain will grow and which will not,
Speak then to me, who neither beg nor fear
Your favours nor your hate.

– Macbeth (Banquo)

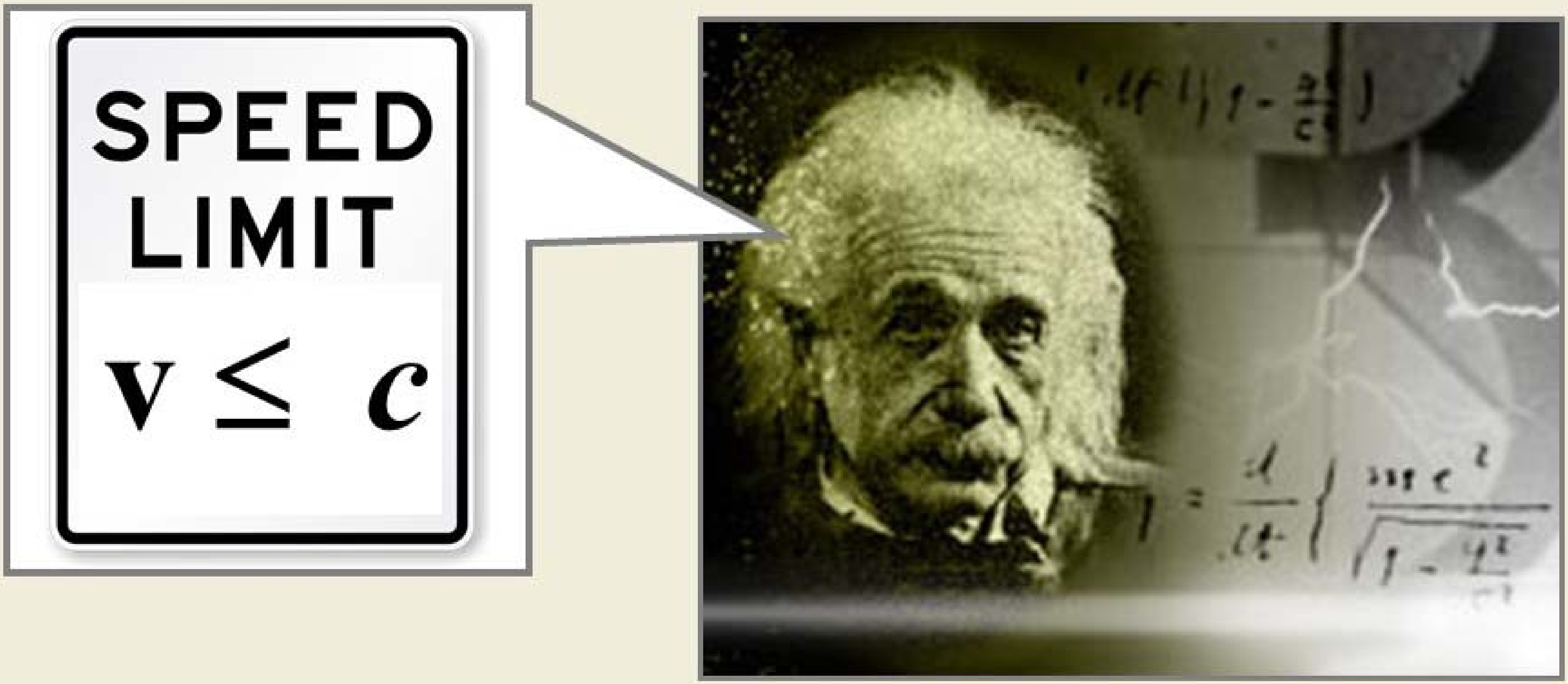
Primordial Seeds of Structure



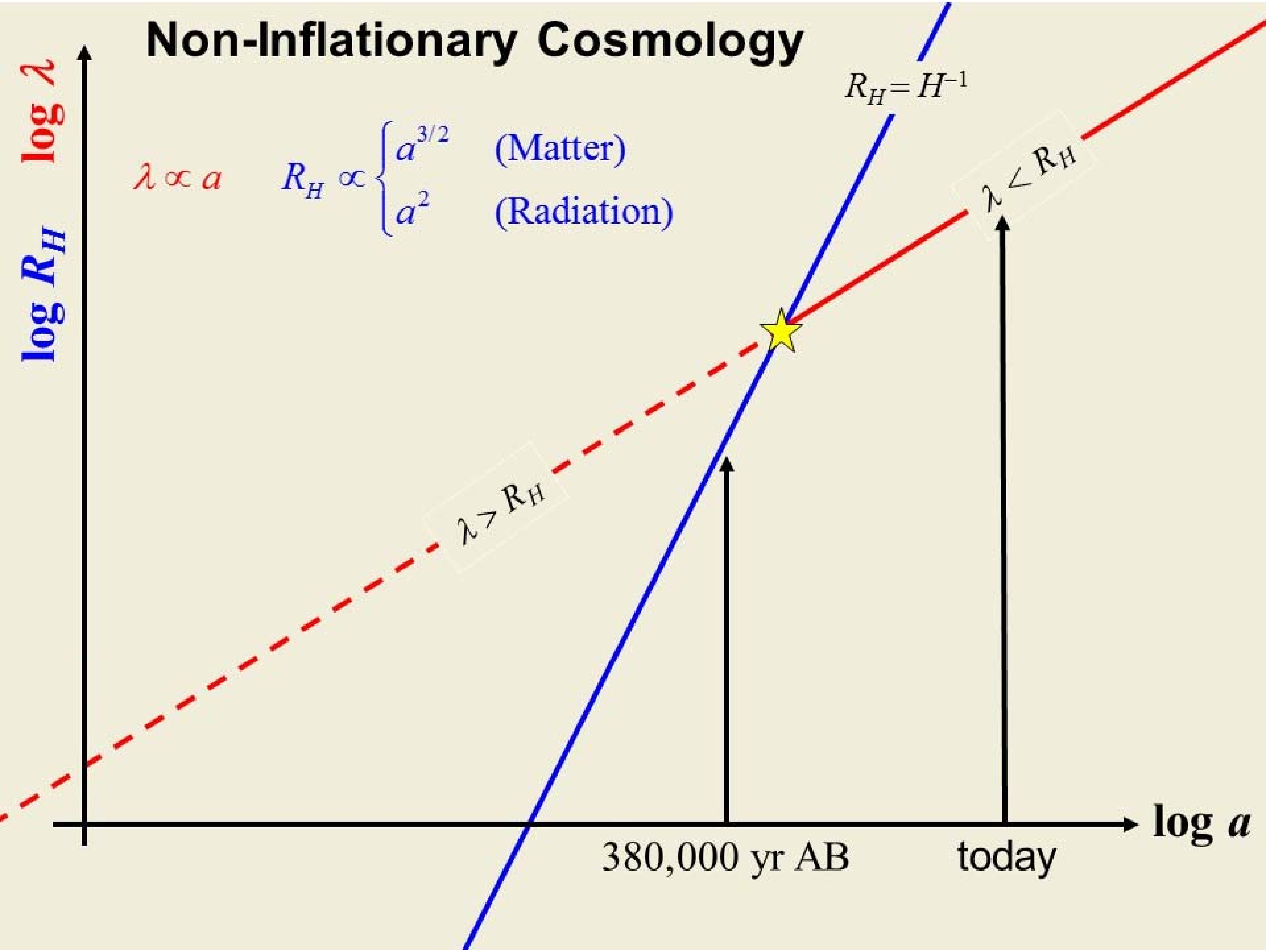


correlations on scales $\gg$ 380,000 light years

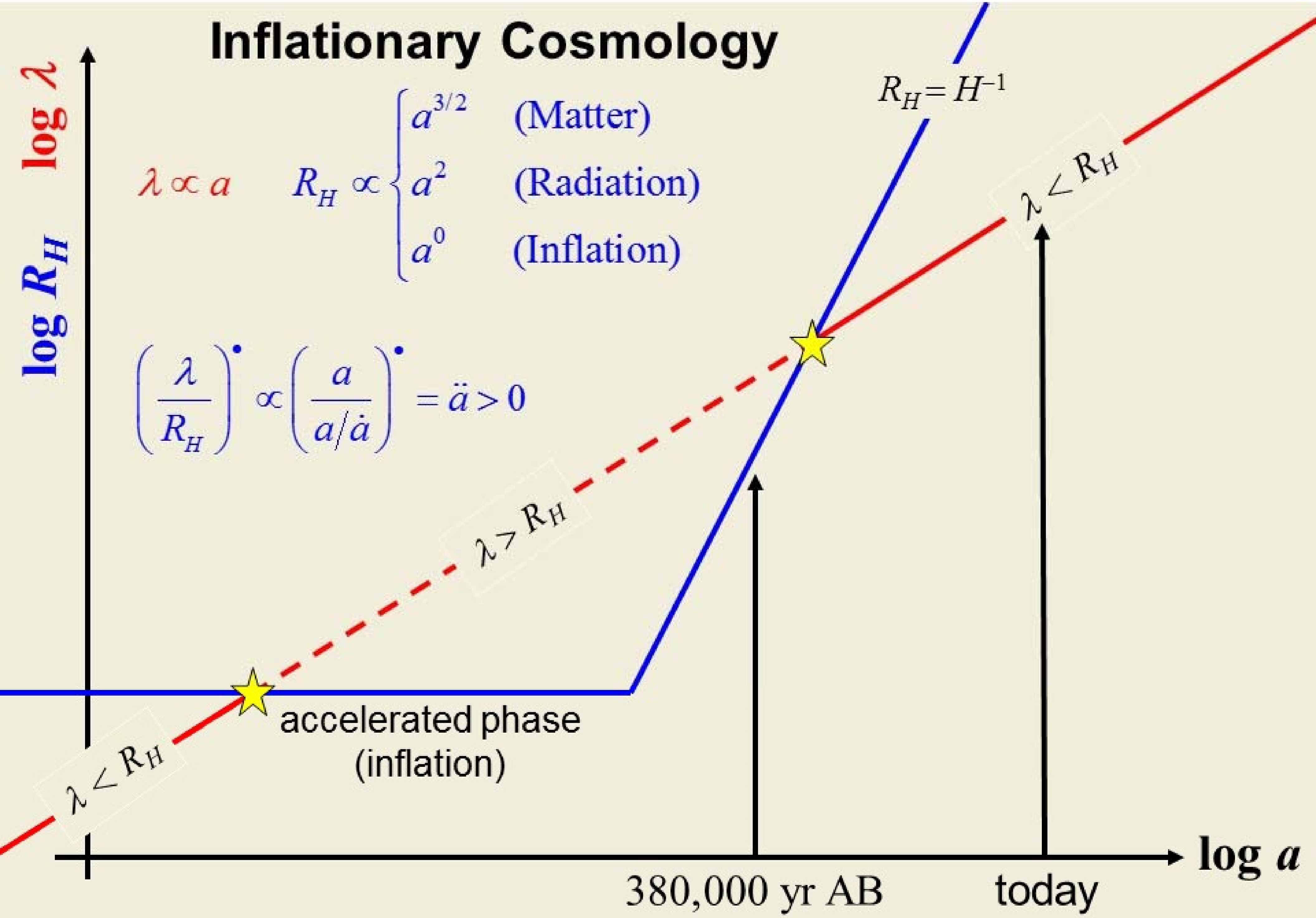
More Than 380,000 Light Years In Less Than 380,000 Years?



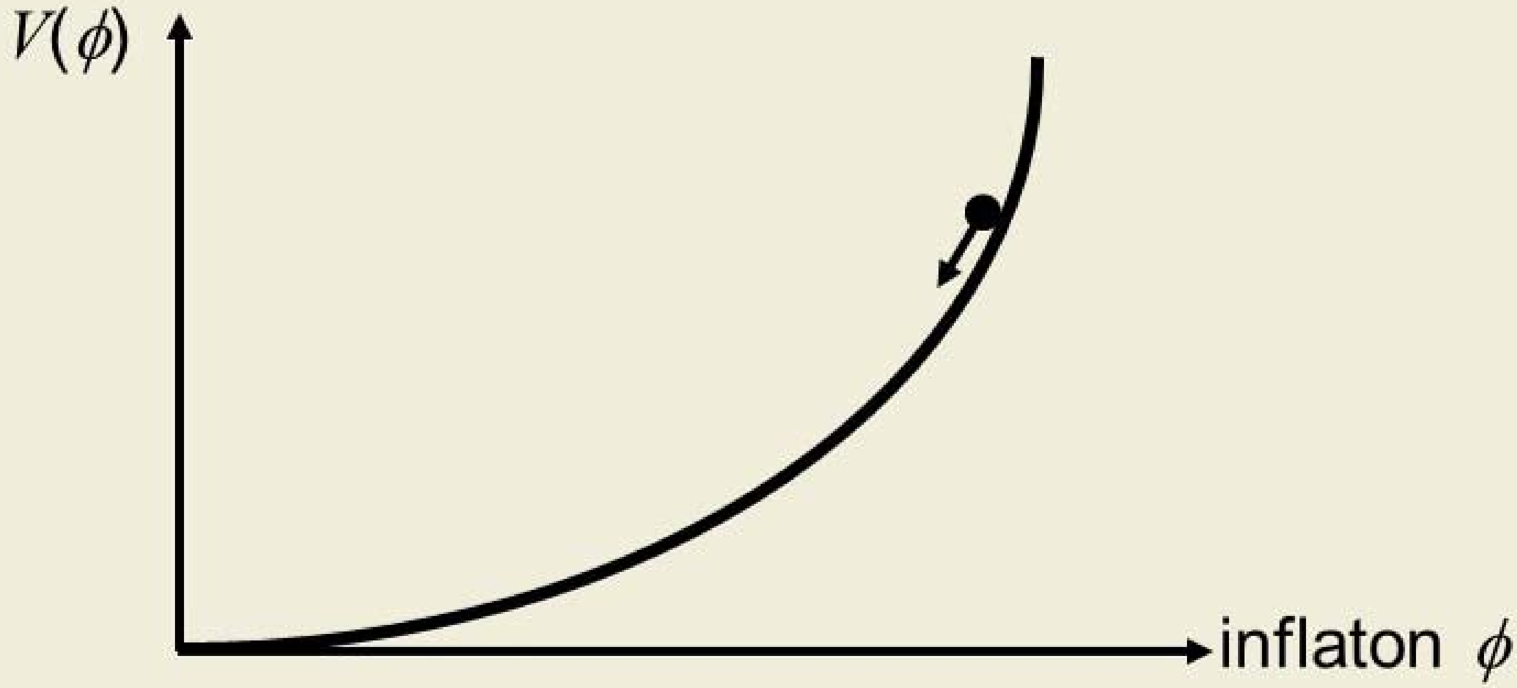
- $v \leq c$ for velocity <u>*through*</u> space
- no limit on expansion velocity <u>*of*</u> space
- “acausal” requires “accelerated” expansion



Non-Inflationary Cosmology
log λ
log R_H
λ ∝ a
R_H ∝ { a^{3/2} (Matter); a^2 (Radiation) }
R_H = H^{-1}
λ < R_H
λ > R_H
log a
380,000 yr AB
today

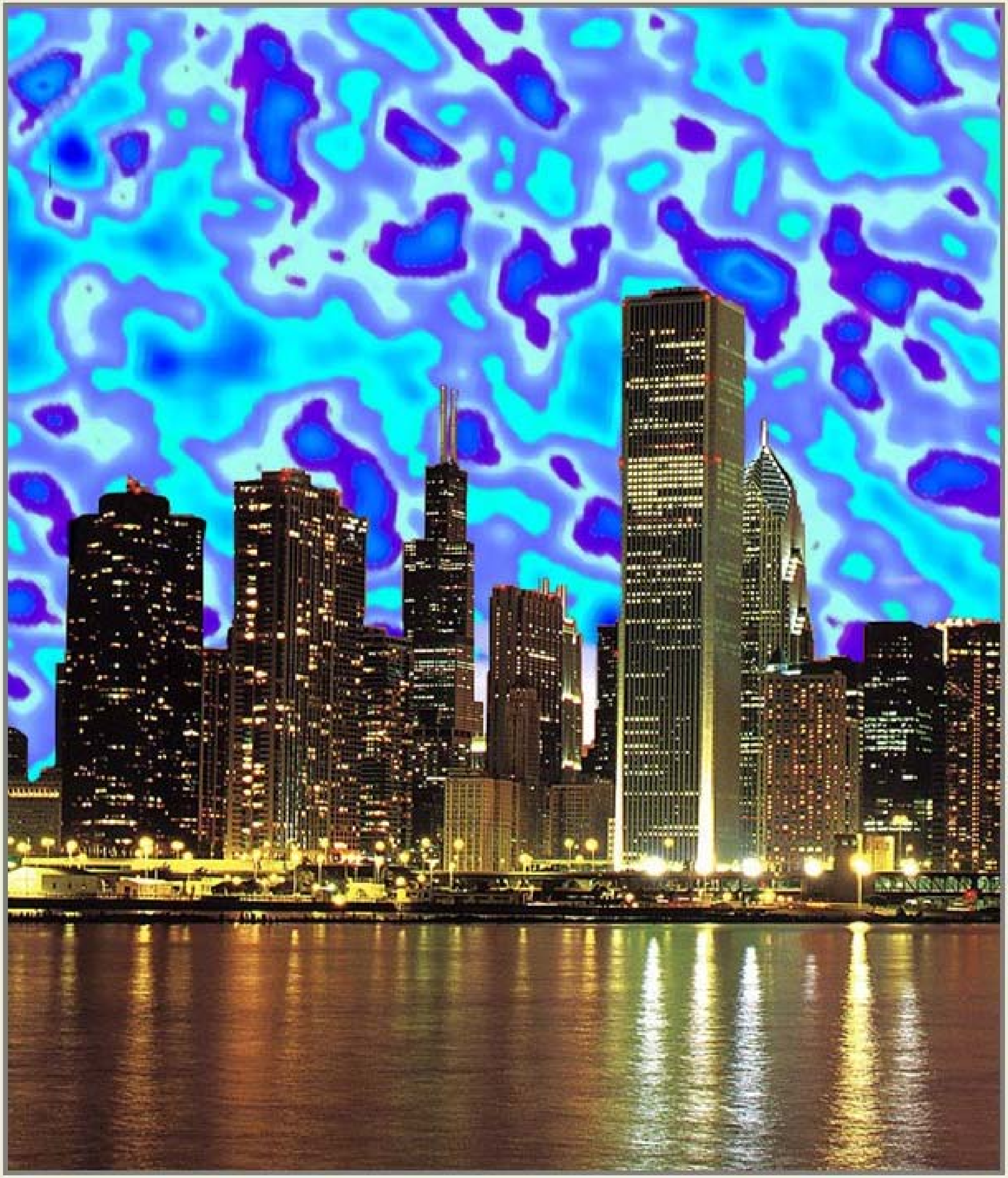

Inflationary Cosmology
log λ
log R_H
$R_H = H^{-1}$
$\lambda \propto a$
$R_H \propto \begin{cases} a^{3/2} & \text{(Matter)} \\ a^2 & \text{(Radiation)} \\ a^0 & \text{(Inflation)} \end{cases}$
$\left(\frac{\lambda}{R_H}\right)^{\bullet} \propto \left(\frac{a}{a/\dot{a}}\right)^{\bullet} = \ddot{a} > 0$
$\lambda < R_H$
$\lambda > R_H$
$\lambda < R_H$
accelerated phase
(inflation)
380,000 yr AB
today
log a

Inflation



- Scalar field has potential energy $V(\phi)$ and kinetic energy $\dot{\phi}^2$
- Potential energy (zero-momentum mode of ϕ) dominates $\Rightarrow$ acceleration
- Schrödinger's Alarming Phenomenon produces inflaton particles ($\hbar \neq 0$)
- Particles produced with non-zero momentum $\delta\phi \rightarrow \delta\rho \rightarrow \delta T$
- Particles produced when they cross Hubble radius during inflation fluctuations on all scales with approximately same amplitude (Harrison—Zel'dovich)

A pattern of vacuum quantum fluctuations



$$\hbar \to 0$$

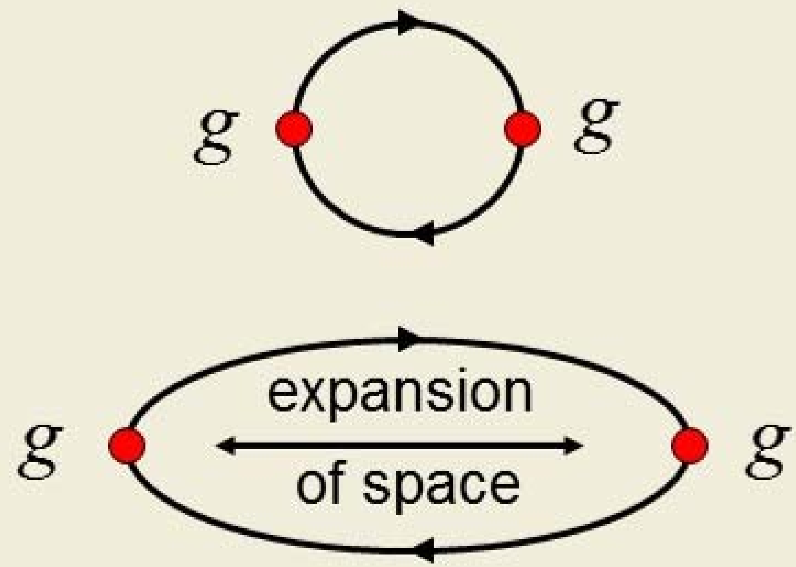
Quantum fluctuations, once microscopic, have been stretched …

… to be as large as the observable universe!

- The map of CMB $\Delta T/T$ is a map of quantum fluctuations
- produced 10^{-35} seconds after the bang during primordial inflation
- when the universe was dominated by vacuum energy
- and rapid expansion ripped particles out of the quantum vacuum
- (Schrödinger’s alarming phenomenon)
- producing primordial seeds of structure that grew to all we see
- (you are an amplified quantum fluctuation),
- and encoded in the pattern is the imprint of fundamental physics.

Disturbing the Quantum Vacuum

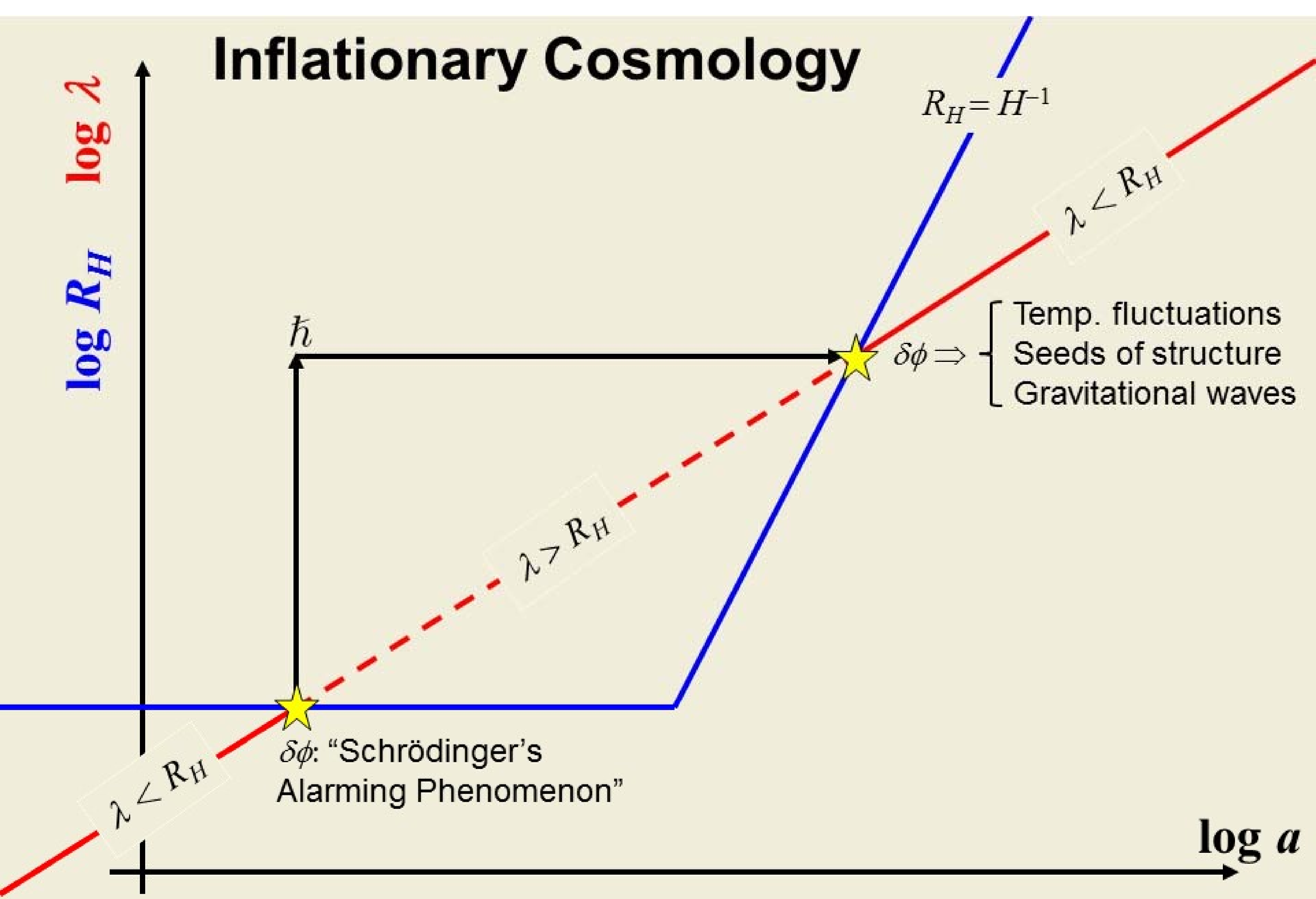
Expanding Universe ⟶ Particle creation

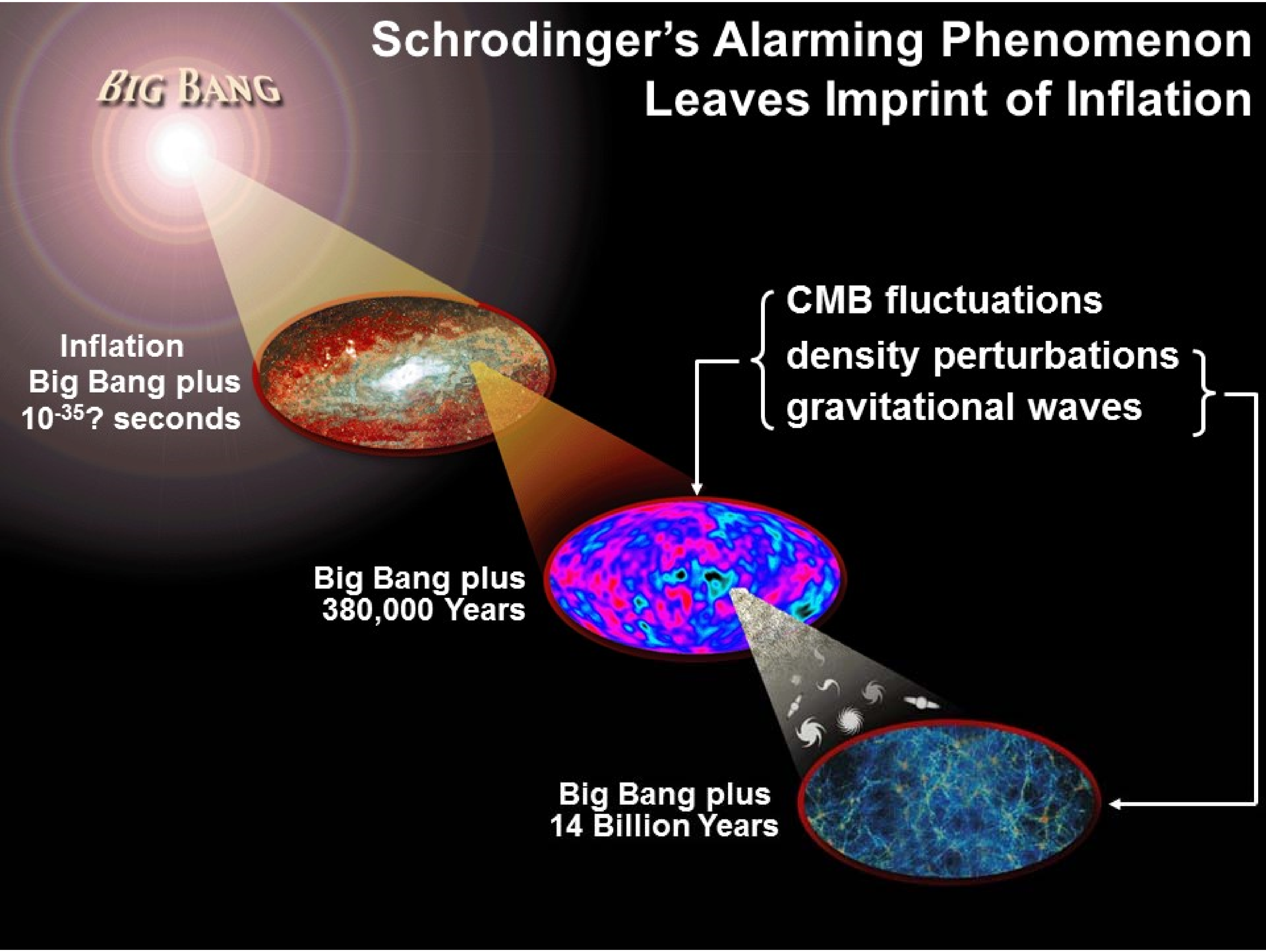


Particle creation if energy gained in expansion over a Compton wavelength exceeds the particle's rest mass.

Schrödinger's Alarming Phenomenon applies to gravitons

Inflationary Cosmology



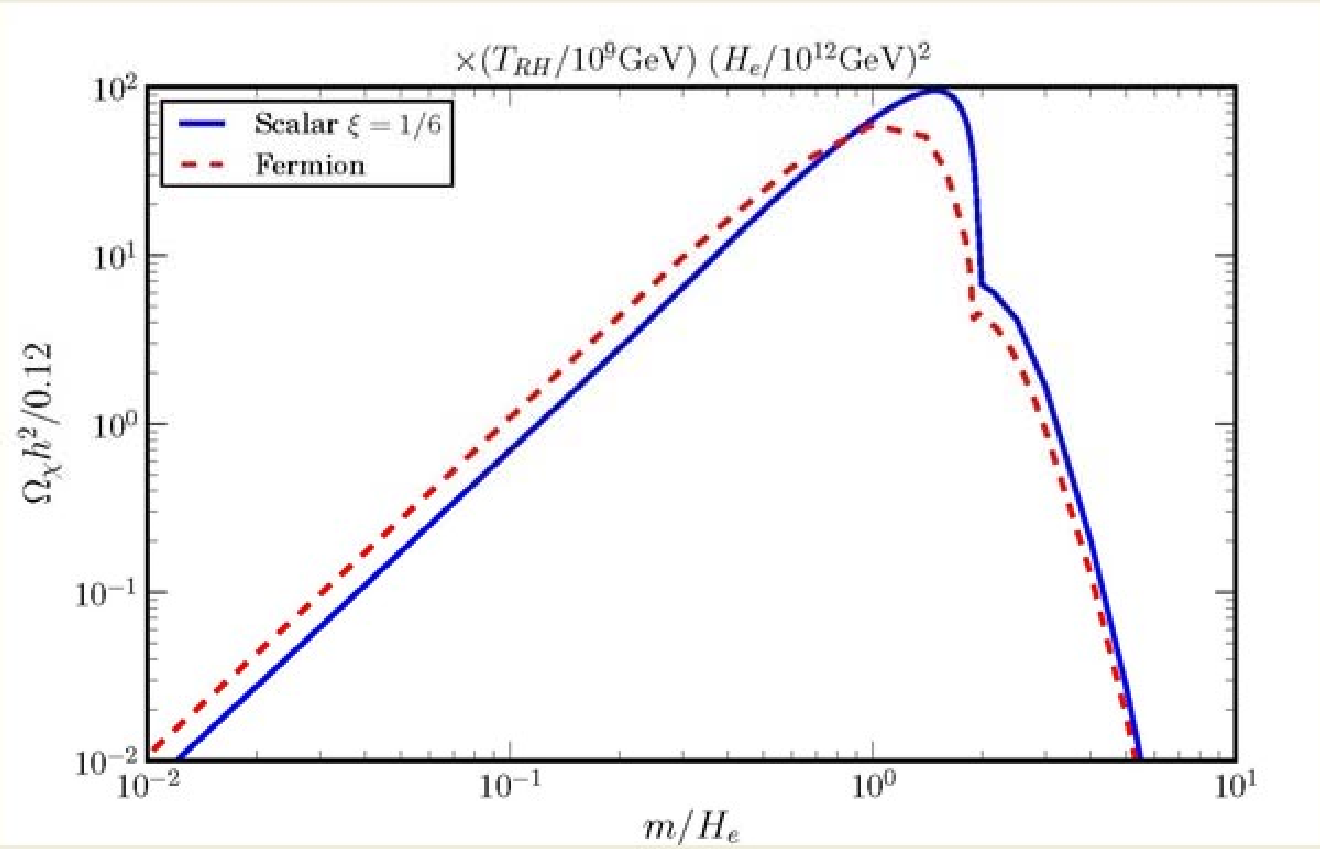


Schrodinger's Alarming Phenomenon
Leaves Imprint of Inflation
BIG BANG
Inflation
Big Bang plus
10-35? seconds
CMB fluctuations
density perturbations
gravitational waves
Big Bang plus
380,000 Years
Big Bang plus
14 Billion Years

WIMPzillas

Chung, Kolb, Riotto; Kuzmin, Tkachev; .... Kolb & Long

- Schrödinger's Alarming Phenomenon applies to all particles*
- All particles* produced during inflation
- Could dark matter be produced during inflation?



- Inflation signifies a new mass scale.
- H_e, expansion rate at end of inflation, comparable to inflaton mass.
- Might expect other particles with mass comparable to inflaton mass.
- If one of them stable, natural candidate for dark matter (WIMPzilla miracle).

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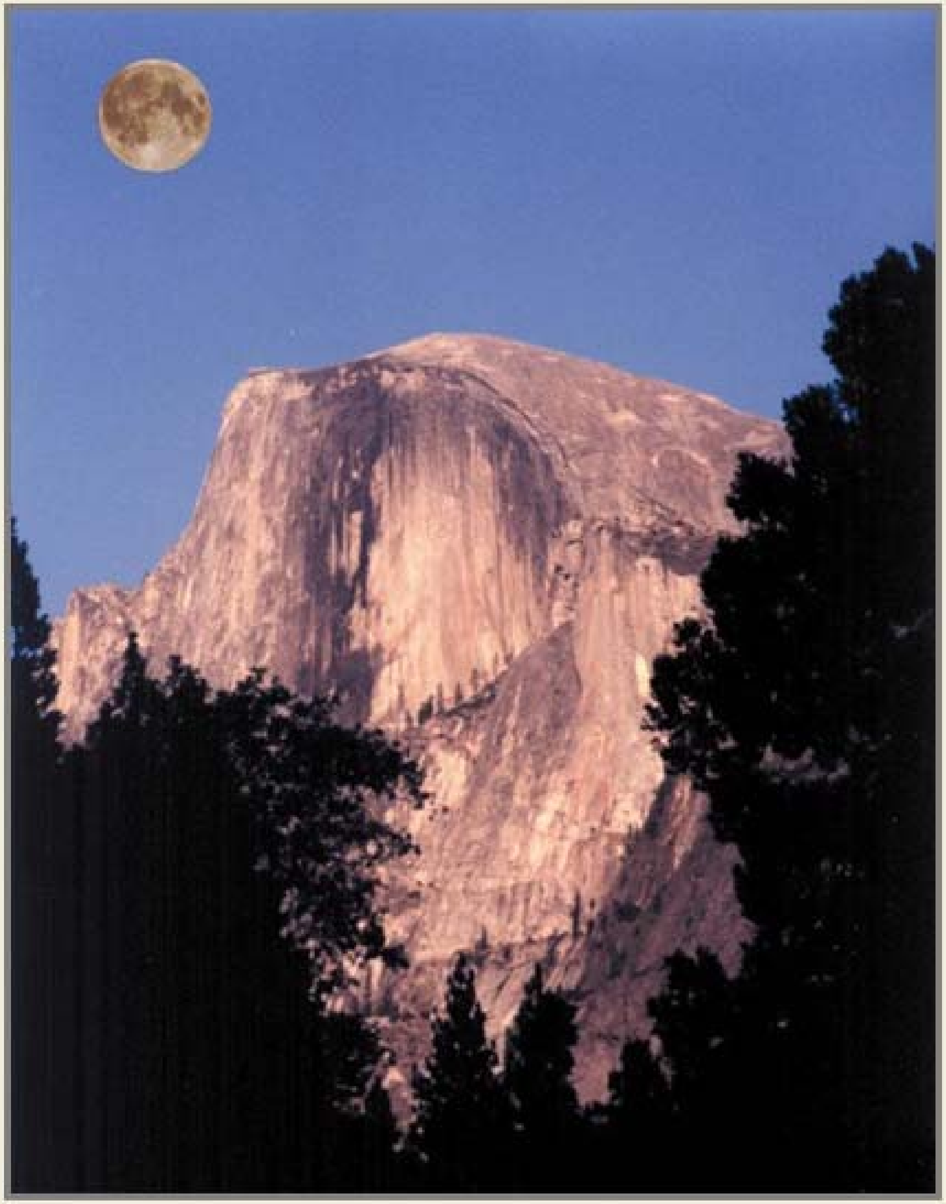
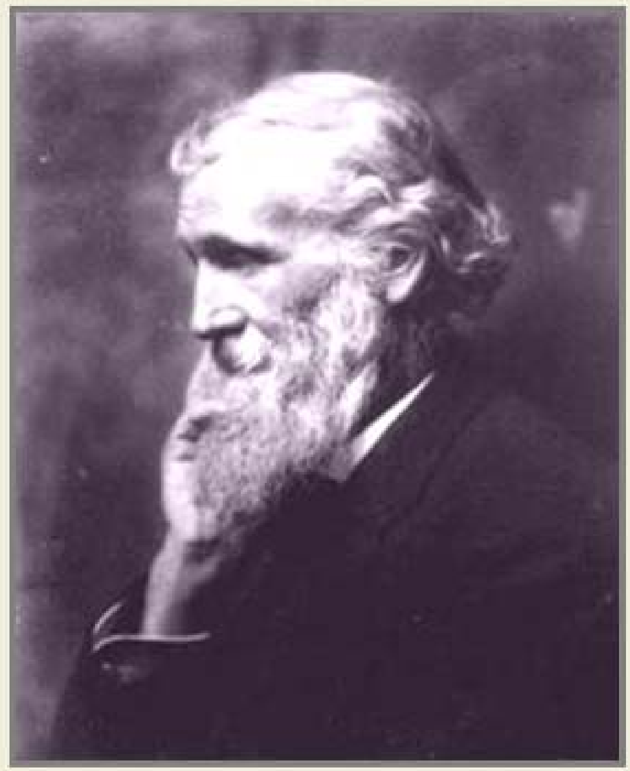
* So long as conformal symmetry violated.

Schrodinger's Alarming Phenomenon Connects the Quantum & the Cosmos



When we try to pick out anything by itself, we find it bound fast, by a thousand invisible cords that cannot be broken, to everything else in the universe.

– John Muir



ERWIN SCHRÖDINGER'S ALARMING PHENOMENON

Schrödinger's 1939 deep insight into particle creation in the expanding universe

- was correct in principle (although some technical missteps)
- was not then (and isn't much now) appreciated
- was alarming to Schrödinger but now a fundamental part of cosmology
- has profound implication for understanding our present universe
  - we are all amplified quantum fluctuations!
  - gravitons?
  - dark matter?
  - other?

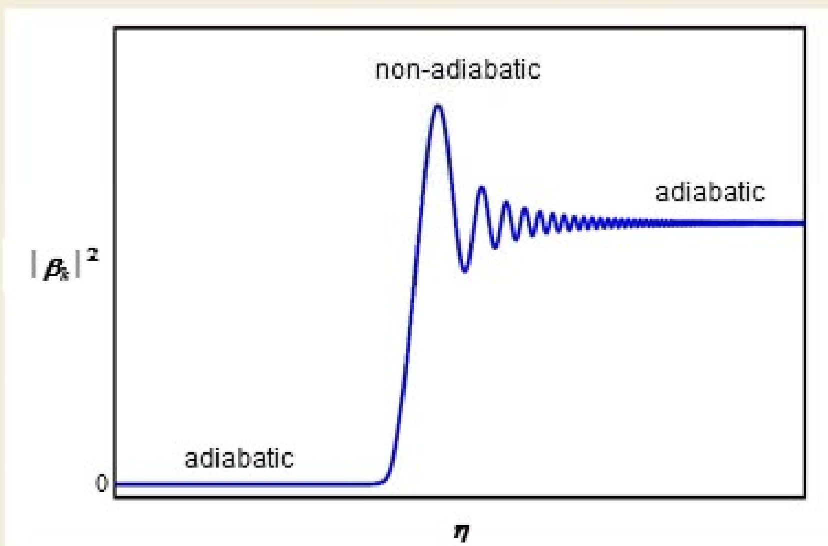
Lessons to be learned

- lower the anchor of our peaceful studies into the ground of eternity
- don't be alarmed
- trust your equations
- beautiful physics has far-reaching implications

ERWIN SCHRÖDINGER'S ALARMING PHENOMENON

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

The Proper Vibrations of the Expanding Universe



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